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Boundary effects on Čerenkov radiation in an alternating electric field

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Abstract. The effects of a boundary separating two dielectrics on the radiation due to a relativistic charged particle moving with a uniform linear superphase velocity through one of them, at a height h from the other, are investigated when an alternating electric field is applied first parallel and then perpendicular to the direction of motion of the charge. Due to the presence of the field in this situation, Doppler radiation is emitted, along with the usual Čerenkov radiation. General formulae for the intensities of both types of radiation are derived and analysed, and the Čerenkov radiation is studied in detail for the following cases: (i) when the charge moves in a vacuum at a non-zero height from the boundary; (ii) when it moves exactly along the boundary. In both situations, very compact formulae for the intensity of Čerenkov radiation are obtained. The dependence of Čerenkov radiation on the height h is studied. In the absence of the field, the intensity shows an inverse-square dependence on h. In the second case, the intensity of Čerenkov radiation is found to be nearly 60% of the intensity when the charge moves through an infinite dielectric.

1. Introduction

Ginzburg (1947a, b) has suggested the utilisation of Čerenkov and Doppler effects as one of the possible methods of microwave generation. For the utilisation of the Doppler effect, Ginzburg has considered the passage of non-relativistic electrons near a dielectric of large refractive index when an alternating electric field is applied perpendicular to the direction of motion. In connection with the generation of microwaves, particularly in the region inaccessible by other means, it is of interest to determine the radiation produced by a moving charged particle or a modulated beam close to a dielectric. Motz (1951, 1956, 1968) has examined the radiation from fast-electron beams passing through a succession of electric or magnetic fields of alternating polarity in an arrangement referred to as an 'undulator'. Depending upon the field used to modulate the electron beam, undulators may be electric or magnetic in nature. Magnetic undulators are given much more attention in the literature than the electric ones. The possibility of using a vacuum undulator to measure the total energy of an individual particle in the ultra-relativistic region is indicated by Motz (1956) and Korkhmazyan (1970, 1972). Ginzburg (1972) has pointed out that when a transparent medium is present in the undulator, the radiation intensity increases sharply under certain conditions. Radiation from relativistic particles in an undulator is analysed by Alferov *et al* (1973, 1974).

In connection with the development of relativistic electron accelerators (betatron, synchrotron etc), research has been devoted to the question of emission from a relativistic electron moving in a magnetic field by a number of workers (Arcimovich and Pomeranchuk 1946, Schiff 1946, Schott 1912, Klepikov 1954, Sokolov and Matveev 1956). The intensity of radiation of a charged particle moving in a constant electric or magnetic field, and its spectral and angular distributions, have been thoroughly described (Landau and Lifshitz 1971, Jackson 1962, Ginzburg *et al* 1968, Sokolov and Ternov 1968). The results obtained for emission of radiation from an accelerated charge in a vacuum have been generalised for the motion of a charged particle in a medium. For the case of motion in a circle this is done by Tsytovich (1951), Ter-Mikaelyan (1959) and Kitao (1960), while the case of helical motion is considered by Kukanov *et al* (1971) and Andreev (1972). The law of motion of a charge moving in an isotropic transparent medium in the field of an intense monochromatic electromagnetic wave is found by Dement'ev *et al* (1972). The problem of emission by charged particles in the field of a plane electromagnetic wave in a medium with a refractive index $n_0 > 1$ is tackled by Arutyunyan and Avetisyan (1972).

With a view to generating microwaves, it is of interest to discuss the effects of electromagnetic fields on Čerenkov radiation. The problem of Čerenkov radiation in an alternating electric field is considered by Diasamidze and Tavdgiridze (1972). Changes in Čerenkov radiation caused by an external field have been studied by Mysakhanyan and Nikishov (1974). They have considered helical motion of an electron in a medium and the motion of a charge in the field of a plane electromagnetic wave, and have discussed the conditions for a field to affect Čerenkov radiation qualitatively. The effects of an alternating electric field on Čerenkov radiation in an infinite isotropic dielectric medium have been considered by Risbud and Takwale (1977, 1979).

In the situation considered by Risbud and Takwale (1977, 1979), i.e. a charge moving through an infinite dielectric, the energy lost by the charge through ionisation, excitation and atomic collisions etc is quite large compared to the energy loss due to Čerenkov radiation. With a view to minimising the energy losses of the charge by processes other than the Čerenkov effect, the situation when the charge is moving close to the surface of a dielectric is worth considering. From this point of view the problem of Čerenkov radiation from electrons moving parallel to the surface of a semi-infinite plane dielectric has been treated by several authors: Danos (1955), Linhart (1955), Bogdankevich and Bolotovsky (1957), Sitenko and Tkalich (1960), Garibyan and Mergelyan (1960) and Thomas (1972).

The aim of the present paper is to study the effects of an alternating electric field on Čerenkov radiation when a charge is moving above a boundary separating two dielectrics. The same problem is discussed in part by Diasamidze and Tavdgiridze (1972), giving the following conclusions. The alternating electric field of frequency ω_0 can have a strong effect on Čerenkov radiation and leads to the appearance of Doppler radiation at a frequency greater than ω_0. At small amplitudes of the electric field, the Čerenkov radiation is reduced slightly and the Doppler radiation increases as the square of the field amplitude. A marked reduction in the energy losses in Čerenkov radiation is possible at large amplitudes of the electric field.

We find it difficult to accept the arguments and conclusions of Diasamidze and Tavdgiridze (1972), as their results contain unevaluated integrals which are not in closed form, and hence the evaluation of actual intensities of Čerenkov and Doppler radiations does not seem to be possible from their results. Without evaluating integrals with respect to frequency ω, which is rather a difficult part of the problem, they have drawn, merely on the basis of the integrands, some conclusions regarding the intensities of the two radiations which do not seem to be justifiable. They have considered only extreme cases of fields—weak and strong—and have tried to analyse them qualitatively. Moreover, they have taken only one case, i.e. the alternating electric field applied parallel to the velocity of the charge. In fact the field can be applied in any direction. In particular, the case when the alternating electric field is applied perpendicular to the velocity of the charged particle is quite interesting, and although Ginzburg (1947b) had suggested it as a possible method for utilisation of the Doppler effect for microwave generation, it has not to our knowledge been tackled by any worker so far. As the work of Diasamidze and Tavdgiridze (1972) is rather incomplete, and their conclusions seem to be doubtful, we have undertaken the study of both the cases, with the field parallel and perpendicular to the velocity of the charge, and have derived the results for intensities of Čerenkov and Doppler radiations in closed form. The effects of the boundary on Čerenkov radiation in the presence of parallel as well as perpendicular alternating electric fields are discussed in detail under different physical situations.

2. Parallel field: general expression for the radiation and analysis

Consider a point charge e (figure 1) moving with a uniform linear relativistic superphase velocity $\bar{v}_0$ parallel to the boundary separating two media that are characterised by dielectric constants and magnetic permeabilities ϵ_1 and μ_1 (upper half-space) and ϵ_2 and μ_2 (lower half-space). We assume that the charge moves in the upper medium at a height h from the boundary. An alternating electric field $\boldsymbol{E} = \boldsymbol{E}_0 \,\mathrm{Sin}\, \omega_0 t$ is applied parallel to the direction of velocity of the charge, i.e. along the z axis. The field superimposes oscillations on the uniform linear motion of the charge, so the velocity of the charge $\boldsymbol{v}_0$ changes to $\boldsymbol{v}(t)$, given by the following components:

$$v_x = v_y = 0$$

$$v_z(t) = (v_0 - v'_u \cos \omega_0 t)/[1 - (v_0 v'_u/c^2) \cos \omega_0 t] \tag{1}$$

where

$$\boldsymbol{v}'_u = e\boldsymbol{E}_0/m\omega_0 = (v'_u)_z, \qquad m = m_0/(1-\beta_0^2)^{1/2}, \qquad \beta_0 = |\boldsymbol{v}_0/c|.$$

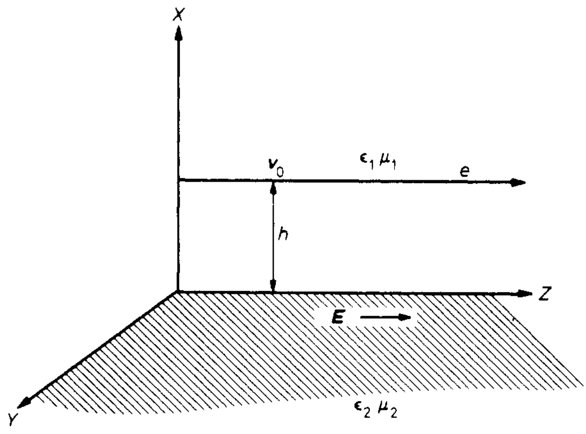

Figure 1. The motion of a charge e with velocity $\boldsymbol{v}_0$ at a height h from the boundary (Y–Z) plane separating two dielectrics characterised by $\epsilon_1\mu_1$ (upper half-space) and $\epsilon_2\mu_2$ (lower half-space).

Consequently, the charge and current densities in Maxwell's equation become

$$\rho(t)=e\delta(x-h)\delta(y)\delta(z-z(t)) \tag{2}$$

$$\boldsymbol{j}(t)=\rho(t)\boldsymbol{v}(t)=j_z(t) \tag{3}$$

where

$$z(t)=\int v_z(t)\,\mathrm{d}t.$$

We assume here that the modulation energy is small compared to the mean particle energy, i.e. $v'_u \ll v_0$, and so we are justified in considering terms linear in v'_u (Risbud and Takwale 1979). Therefore, retaining only such terms, we can write

$$v_z(t)=v_0-v_u(1-\beta_0^2)^{3/2}\cos\omega_0 t \tag{1'}$$

where

$$v_u=eE_0/m_0\omega_0.$$

Using equation (1′), we get

$$z(t)=v_0t-(v_u/\omega_0)(1-\beta_0^2)^{3/2}\sin\omega_0 t. \tag{4}$$

We tackle the problem by the method of images. The boundary surfaces are therefore replaced by proper fictitious image electric and magnetic charge densities. In the case of dynamic charges the method has been developed and applied by Sitenko and Tkalich (1960) and Pafomov (1967). To find fields in the upper half-space (i.e. in dielectric 1) due to the passage of an electron through the dielectric 1 at a height h from the boundary surface separating dielectrics 1 and 2 (figure 1), the entire space is assumed to be filled by dielectric 1 and, in addition to the charge e located at time t at the position $(h, 0, v_z(t)\,.\,t)$, there are fictitious electric and magnetic charges of charge densities e_1 and m_1 distributed in the plane $x=-h$, moving with the same velocity as the charge e. The desired field then coincides with that obtained by considering the true charge e along with the image charges e_1 and m_1 in the absence of the boundary. To obtain the fields in the lower half-space, the entire space is assumed to be filled by the dielectric 2, and electric charges of charge density e_2, and magnetic charges of charge density m_2 are assumed to be distributed in the plane $x=h$, moving with the velocity $v_z(t)$. Then the desired field is the same as that produced by the true charge e and the image charge densities e_2 and m_2 in the absence of the boundary. The proper fictitious electric and magnetic charge densities are then determined from the boundary conditions.

2.1. *Field potentials in the upper half-space* $(x>0)$

The electric and magnetic fields in the upper half-space are given by

$$\boldsymbol{E}^{(1)}=\boldsymbol{E}_\mathrm{e}+\boldsymbol{E}_\mathrm{m} \tag{5}$$

$$\boldsymbol{H}^{(1)}=\boldsymbol{H}_\mathrm{e}+\boldsymbol{H}_\mathrm{m} \tag{6}$$

where $\boldsymbol{E}_\mathrm{e}$ and $\boldsymbol{H}_\mathrm{e}$ are the fields associated with the true and fictitious electric charges, while $\boldsymbol{E}_\mathrm{m}$ and $\boldsymbol{H}_\mathrm{m}$ are those associated with the magnetic charges. These are expressed through electric potentials $\boldsymbol{A}$, ϕ and magnetic potentials $\boldsymbol{F}$ and ψ as

$$\begin{aligned}
&\boldsymbol{E}_{\mathrm{e}}=-\nabla\phi-(1/c)(\partial\boldsymbol{A}/\partial t)\\
&\boldsymbol{H}_{\mathrm{e}}=(1/\mu)\nabla\times\boldsymbol{A}\\
&\boldsymbol{E}_{\mathrm{m}}=(-1/\epsilon)\nabla\times\boldsymbol{F}\\
&\boldsymbol{H}_{\mathrm{m}}=-\nabla\psi-(1/c)(\partial\boldsymbol{F}/\partial t).
\end{aligned}\tag{7}$$

Substituting fields from equations (5), (6) and (7) in Maxwell's equations and using the Lorentz gauge, we obtain the following equations for the field potentials:

$$\begin{aligned}
&\nabla^2\boldsymbol{A}-(\epsilon_1\mu_1/c^2)\partial^2\boldsymbol{A}/\partial t^2=-(4\pi\mu_1/c)\rho_{\mathrm{e}}(t)\boldsymbol{v}(t)\\
&\nabla^2\phi=(\epsilon_1\mu_1/c^2)\partial^2\phi/\partial t^2=-(4\pi/\epsilon_1)\rho_{\mathrm{e}}(t)\\
&\nabla^2\boldsymbol{F}-(\epsilon_1\mu_1/c^2)\partial^2\boldsymbol{F}/\partial t^2=-(4\pi\epsilon_1/c)\rho_{\mathrm{m}}(t)\boldsymbol{v}(t)\\
&\nabla^2\psi-(\epsilon_1\mu_1/c^2)\partial^2\psi/\partial t^2=-(4\pi/\mu_1)\rho_{\mathrm{m}}(t)
\end{aligned}\tag{8}$$

where

$$\rho_{\mathrm{e}}(t)=e\delta(x-h)\delta(y)\delta(z-z(t))+e_1(y,z-z(t))\delta(x+h)$$

and

$$\rho_{\mathrm{m}}(t)=m_1(y,z-z(t))\delta(x+h)$$

are respectively the densities of electric and magnetic charges. The second term in $\rho_{\mathrm{e}}(t)$ determines the incremental charge e_1 (of the image) distributed in the plane $x=-h$. We find the solutions of equations (8) in the form

$$Q(\boldsymbol{r},t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}Q(x,k_y,k_z,\omega)\exp[\mathrm{i}(k_y y+k_z z-\omega t)]\,\mathrm{d}k_y\,\mathrm{d}k_z\,\mathrm{d}\omega\tag{9}$$

where the function $Q(\boldsymbol{r},t)$ represents any one of the field potentials $\boldsymbol{A}(\boldsymbol{r},t)$, $\phi(\boldsymbol{r},t)$, $\boldsymbol{F}(\boldsymbol{r},t)$ and $\psi(\boldsymbol{r},t)$.

Using a property of Bessel functions:

$$\exp(\mathrm{i}a\sin\omega_0 t)=\sum_{l=-\infty}^{+\infty}J_l(a)\exp(\mathrm{i}l\omega_0 t)$$

and substituting the potentials from equation (9) in equation (8), we get the following equations for the expansion coefficients:

$\phi(x,k_y,k_z,\omega)$, $A_z(x,k_y,k_z,\omega)$, $\psi(x,k_y,k_z,\omega)$, $F_z(x,k_y,k_z,\omega)$.

$$\begin{aligned}
\frac{\partial^2\phi}{\partial x^2}-\chi_1^2\phi=\frac{e}{\pi\epsilon_1}\delta(x-h)&\sum_{l=-\infty}^{+\infty}J_l\left(\frac{k_z v_u(1-\beta_0^2)^{3/2}}{\omega_0}\right)\\
&\times\delta(\omega+l\omega_0-k_z v_0)-\frac{e_1(k_y,k_z,\omega)}{2\pi^2\epsilon_1}\delta(x+h)
\end{aligned}\tag{10}$$

$$\begin{aligned}
\frac{\partial^2 A_z}{\partial x^2}-\chi_1^2 A_z=\frac{-\mu_1 e v_z(t)}{\pi c}\delta(x-h)&\sum_{l=-\infty}^{+\infty}J_l\left(\frac{k_z v_u(1-\beta_0^2)^{3/2}}{\omega_0}\right)\\
&\times\delta(\omega+l\omega_0-k_z v_0)-\frac{\mu_1 v_z(t)}{2\pi^2 c}\delta(x+h)e_1(k_y,k_z,\omega)
\end{aligned}\tag{11}$$

$$\partial^2\psi/\partial x^2 - \chi_1^2\psi = -(m_1(k_y, k_z, \omega)/2\pi^2\mu_1)\delta(x+h) \tag{12}$$

$$\partial^2 F_z/\partial x^2 - \chi_1^2 F_z = -(\epsilon_1 v_z(t)/2\pi^2 c)\delta(x+h)m_1(k_y, k_z, \omega) \tag{13}$$

where

$$\chi_1^2 = k_y^2 + k_z^2 - \omega^2\epsilon_1\mu_1/c^2 \tag{14}$$

$$e_1(k_y, k_z, \omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e_1(y, z - z(t)) \exp[-\mathrm{i}(k_y y + k_z z - \omega t)]\, \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}t \tag{15}$$

and a similar expression for $m_1(k_y, k_z, \omega)$.

For a dispersive medium both the dielectric constant ϵ and magnetic permeability μ are functions of frequency. The solutions of equations (10), (11), (12) and (13), which vanish at infinity can be obtained by the Green's function method used in Davydov (1965). These are given by

$$\begin{aligned}
\phi^{(1)}(x, k_y, k_z, \omega) &= \frac{e}{2\pi\epsilon_1}\frac{\exp(-\chi_1|x-h|)}{\chi_1}\sum_{l=-\infty}^{+\infty} J_l\left(\frac{k_z v_u(1-\beta_0^2)^{3/2}}{\omega_0}\right) \\
&\quad \times \delta(\omega + l\omega_0 - k_z v_0) + \frac{e_1}{4\pi^2\epsilon_1}\frac{\exp[-\chi_1(x+h)]}{\chi_1} \\
A_z^{(1)}(x, k_y, k_z, \omega) &= \beta_z n_1^2 \phi^{(1)}(x, k_y, k_z, \omega) \\
\psi^{(1)}(x, k_y, k_z, \omega) &= \frac{m_1}{4\pi^2\mu_1}\frac{\exp[-\chi_1(x+h)]}{\chi_1} \\
F_z^{(1)}(x, k_y, k_z, \omega) &= \beta_z n_1^2 \psi^{(1)}(x, k_y, k_z, \omega)
\end{aligned} \tag{16}$$

where

$$x > 0, \qquad \operatorname{Re}\chi_1 < 0, \qquad \beta_z = v_z(t)/c, \qquad n_1^2 = \epsilon_1\mu_1.$$

2.2. Field potentials in the lower half-space ($x < 0$)

In the lower half-space, we determine the electromagnetic field using the electric and magnetic charge densities as

$$\rho_e(t) = e_2(y, z - z(t))\delta(x-h)$$

and

$$\rho_m(t) = m_2(y, z - z(t))\delta(x-h).$$

In this case the expansion coefficients for the electric and magnetic potentials are

$$\begin{aligned}
\phi^{(2)}(x, k_y, k_z, \omega) &= (e_2/4\pi^2\epsilon_2)\exp[-\chi_2(h-x)]/\chi_2 \\
A_z^{(2)}(x, k_y, k_z, \omega) &= \beta_z n_2^2 \phi^{(2)}(x, k_y, k_z, \omega) \\
\psi^{(2)}(x, k_y, k_z, \omega) &= (m_2/4\pi^2\mu_2)\exp[-\chi_2(h-x)]/\chi_2 \\
F_z^{(2)}(x, k_y, k_z, \omega) &= \beta_z n_2^2 \psi^{(2)}(x, k_y, k_z, \omega)
\end{aligned} \tag{17}$$

where

$$\chi_2^2 = k_y^2 + k_z^2 - \omega^2 n_2^2/c^2, \qquad n_2^2 = \epsilon_2\mu_2, \qquad \operatorname{Re}\chi_2 > 0, \qquad x < 0.$$

2.3. *Electromagnetic field of the charge*

We can now determine the electric potentials $\phi(\boldsymbol{r}, t)$, $A_z(\boldsymbol{r}, t)$ and the magnetic potentials $\psi(\boldsymbol{r}, t)$ and $F_z(\boldsymbol{r}, t)$ in the two media by equations (9), (16) and (17). Using equations (7), (5) and (6) the electric and magnetic fields can be written as

$$\boldsymbol{E} = -\nabla\phi - (1/c)\partial\boldsymbol{A}/\partial t - (1/\epsilon)\nabla\times\boldsymbol{F}$$

and

$$\boldsymbol{H} = -\nabla\psi - (1/c)\partial\boldsymbol{F}/\partial t + (1/\mu)\nabla\times\boldsymbol{A}.$$

Writing these fields in component form and applying the boundary conditions at $x = 0$, i.e.

$$E_z^{(1)} = E_z^{(2)}, \qquad E_y^{(1)} = E_y^{(2)}, \qquad H_z^{(1)} = H_z^{(2)}, \qquad H_y^{(1)} = H_y^{(2)},$$

we obtain the following four equations for the electromagnetic potentials:

$$\begin{aligned}
&(\partial/\partial z)(\phi^{(1)} - \phi^{(2)})|_{x=0} = (\beta_z/c)(\partial/\partial t)(n_2^2\phi^{(2)} - n_1^2\phi^{(1)})|_{x=0}\\
&(\partial/\partial y)(\phi^{(2)} - \phi^{(1)})|_{x=0} = \beta_z(\partial/\partial x)(\mu_2\psi^{(2)} - \mu_1\psi^{(1)})|_{x=0}\\
&(\partial/\partial z)(\psi^{(1)} - \psi^{(2)})|_{x=0} = (\beta_z/c)(\partial/\partial t)(n_2^2\psi^{(2)} - n_1^2\psi^{(1)})|_{x=0}\\
&(\partial/\partial y)(\psi^{(1)} - \psi^{(2)})|_{x=0} = \beta_z(\partial/\partial x)(\epsilon_2\phi^{(2)} - \epsilon_1\phi^{(1)})|_{x=0}.
\end{aligned} \tag{18}$$

Substituting the solutions of electromagnetic potentials given by equations (16), (17) and (9) and putting $x = 0$ in equations (18), we get four equations for the four unknowns e_1, m_1, e_2 and m_2. Taking the dominant terms $\sim\beta_0$ only (as $v_u \ll v_0$, we are justified in doing a linear approximation in v_u (Risbud and Takwale 1979)) and solving the four equations simultaneously, we determine the fictitious charge densities

$$e_1(k_y, k_z, \omega), \qquad m_1(k_y, k_z, \omega), \qquad e_2(k_y, k_z, \omega), \qquad m_2(k_y, k_z, \omega).$$

Here we give only the expression for $e_1(k_y, k_z, \omega)$ since it is sufficient for the determination of the energy losses.

$$\begin{aligned}
e_1(k_y, k_z, \omega) = 2\pi e &\sum_{l=-\infty}^{+\infty} J_l\left(\frac{k_z v_u(1-\beta_0^2)^{3/2}}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_0)\\
&\times\left\{\frac{k_y^2(1-\alpha)^2 + \beta_0^2(\mu_1\chi_1 + \alpha\mu_2\chi_2)(\epsilon_1\chi_1 - \alpha\epsilon_2\chi_2)}{-k_y^2(1-\alpha)^2 + \beta_0^2(\mu_1\chi_1 + \alpha\mu_2\chi_2)(\epsilon_1\chi_1 + \alpha\epsilon_2\chi_2)}\right\}
\end{aligned} \tag{19}$$

where

$$\alpha = (1 - n_1^2\omega^2/c^2k_z^2)/(1 - n_2^2\omega^2/c^2k_z^2).$$

In the absence of a field (i.e. $v_u = 0$) the image charge e_1 given by equation (19) reduces to equation (10) of Sitenko and Tkalich (1960).

2.4. *Energy radiated*

When a charge moves through a medium, it is acted upon by a Lorentz force given by

$$\boldsymbol{F} = e[\boldsymbol{E} + (\boldsymbol{v}/c)\times\boldsymbol{B}] \qquad (\boldsymbol{B} = \mu\boldsymbol{H}).$$

The total energy lost by the charge per unit length of path is determined by the projection of this force, taken with opposite sign, on the direction of motion of the charge, i.e. the problem reduces to the calculation of the electric field at the point at which the charge is situated. Furthermore, the energy lost by the charge per unit path length is the same as the energy given out in radiation. Hence

$$-\partial\epsilon/\partial z = -eE_z|_{x=h, y=0, z=z(t)}.$$

Using equations (5), (7), (9) and (16), we obtain the following expression for the z component of the electric field:

$$E_z = \frac{-\mathrm{i}}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\mathrm{d}k_y\,\mathrm{d}k_z\,\mathrm{d}\omega}{\epsilon_1\chi_1}k_z\left(1-\frac{\omega v_z n_1^2}{k_z c^2}\right)\exp[\mathrm{i}(k_y y + k_z z - \omega t)]$$
$$\times\left\{e\exp[-\chi_1|x-h|]\sum_{l=-\infty}^{+\infty}J_l\left(\frac{k_z v_u(1-\beta_0^2)^{3/2}}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_0)\right.$$
$$\left.+\frac{e_1(k_y, k_z, \omega)}{2\pi}\exp[-\chi_1(x+h)]\right\}$$

where $e_1(k_y, k_z, \omega)$ is given by equation (19).

We can now write the following expression for the total energy lost by the charge e moving with a uniform superphase velocity through medium ϵ_1 at a height h above the boundary with medium ϵ_2 in the presence of a parallel alternating electric field:

$$-\frac{\partial\epsilon}{\partial z} = \mathrm{Re}\frac{\mathrm{i}e^2}{2\pi}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}k_y\,\mathrm{d}k_z\,\mathrm{d}\omega\frac{k_z}{\epsilon_1\chi_1}\left(1-\frac{n_1^2\omega v_z}{k_z c^2}\right)$$
$$\times J_l^2\left(\frac{k_z v_u(1-\beta_0^2)^{3/2}}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_0)$$
$$\times\left\{1+\exp(-2h\chi_1)\left[\frac{k_y^2(1-\alpha)^2+\beta_0^2(\mu_1\chi_1+\alpha\mu_2\chi_2)(\epsilon_1\chi_1-\alpha\epsilon_2\chi_2)}{-k_y^2(1-\alpha)^2+\beta_0^2(\mu_1\chi_1+\alpha\mu_2\chi_2)(\epsilon_1\chi_1+\alpha\epsilon_2\chi_2)}\right]\right\}. \tag{20}$$

Equation (20) gives Čerenkov radiation for $l=0$ while it corresponds to Doppler radiation for $l\neq 0$. It consists of two parts. The first part, which corresponds to the figure 1 in the braces, represents the total energy loss of a point charge which moves with a superphase velocity through an infinite isotropic medium characterised by constants ϵ_1 and μ_1 in the presence of a parallel alternating electric field. The second term in the braces of equation (20) represents the contribution to the energy loss due to the presence of another dielectric characterised by ϵ_2, μ_2 when a parallel alternating electric field is applied.

We therefore write symbolically

$$-\partial\mathscr{E}/\partial z = -\partial\mathscr{E}^{\mathrm{unbd}}/\partial z - \partial\mathscr{E}^{\mathrm{bd}}/\partial z$$

where $-\partial\mathscr{E}^{\mathrm{unbd}}/\partial z$ represents the energy loss in an unbounded dielectric while $-\partial\mathscr{E}^{\mathrm{nd2}}\partial z$ represents the effects of a boundary. Let us consider these two parts separately.

The integration with respect to k_z in equation (20) can be performed using a δ function. For integration with respect to k_y we use

$$k_y \leqslant (\omega + l\omega_0/v_0)[\omega^2 n_1^2 \beta_0^2/(\omega + l\omega_0)^2 - 1]^{1/2}$$

which is obtained as as a result of the conditions

$$\operatorname{Re} \chi_1 < 0, \qquad \omega^2 n_1^2 \beta_0^2/(\omega + l\omega_0)^2 > 1.$$

Performing integration with respect to k_z and k_y for the first term within the braces of equation (20), we obtain the following expression for the energy loss in an unbounded medium:

$$-\frac{\partial \mathscr{E}^{\text{unbd}}}{\partial z} = -\left(\frac{e^2 \mu_1}{c^2}\right) \sum_{l=-\infty}^{+\infty} \int_0^\infty \mathrm{d}\omega (\omega + l\omega_0)\left(\frac{\omega}{\omega + l\omega_0} - \frac{1}{\beta_0^2 n_1^2}\right)$$
$$\times J_l^2\left(\frac{(\omega + l\omega_0) v_u (1-\beta_0^2)^{3/2}}{\omega_0 v_0}\right). \tag{21}$$

Equation (21) agrees with equation (17) of Risbud and Takwale (1977), except for the factor $(1-\beta_0^2)^{3/2}$ in the argument of the Bessel function, which arises here as a correction for relativistic electron velocities. We note that by Risbud and Takwale (1977) only dominant terms of the order of v_0 have been taken, and terms of the order of v_u^2 have been neglected without explicit mention. Therefore in equation (17) the multiplying factor v^2 is actually v_0^2. We will not consider equation (21) further, since it is fully analysed by Risbud and Takwale (1977, 1979).

Now let us consider the second term within the braces of equation (20) which gives the effects of the presence of the boundary at $x = 0$, in the following two interesting situations. Let us put $l = 0$ in equation (20), so that the mathematics becomes simple and manageable. Moreover, whatever applies to Čerenkov radiation can be extended to Doppler radiation.

2.5. Radiation when the charge moves in a vacuum at a height h from the surface of a dielectric

Here

$$\epsilon_1 = 1, \qquad \epsilon_2 = \epsilon, \qquad \mu_1 = \mu_2 = 1, \qquad h \neq 0, \qquad \epsilon\beta_0^2 > 1, \qquad l = 0.$$

Substituting the above values in the second term of equation (20), we get

$$-\partial \epsilon^{\text{bd}}/\partial z = \operatorname{Re}(\mathrm{i}e^2/2\pi) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}k_y\, \mathrm{d}k_z\, \mathrm{d}\omega (k_z/\chi_1)(1 - \omega v_z/k_z c^2)$$
$$\times J_0^2 (k_z v_u (1-\beta_0^2)^{3/2}/\omega_0) \delta(\omega - k_z v_0) \exp(-2h\chi_1)$$
$$\times \left[\frac{k_y^2 (1-\alpha)^2 + \beta_0^2 (\chi_1 + \alpha\chi_2)(\chi_1 - \alpha\epsilon\chi_2)}{-k_y^2(1-\alpha)^2 + \beta_0^2(\chi_1 + \alpha\chi_2)(\chi_1 + \alpha\epsilon\chi_2)}\right]. \tag{22}$$

Integration with respect to k_z in equation (22) can be performed using a δ function. Consider the integral with respect to k_y in equation (22). On account of the exponential factor in the integral, which decreases rapidly with increasing k_y, the main contribution to the integral comes from small values of k_y. Consequently, we can keep only the

$\exp(-2h\chi_1)$ term inside the integral and take the remaining slowly varying function of k_y outside the integral at $k_y = 0$. Therefore from equation (22) we write

$$-\partial\mathscr{E}^{\mathrm{bd}}/\partial z = \mathrm{Re}(\mathrm{i}e^2/2\pi v_0^2)\int_{-\infty}^{+\infty}\omega\,\mathrm{d}\omega(1-\beta_0^2)J_0^2\left(\frac{\omega v_u(1-\beta_0^2)^{3/2}}{\omega_0 v_0}\right)$$
$$\times\left\{\frac{1}{\chi_1}\left[\frac{k_y^2(1-\alpha)^2+\beta_0^2(\chi_1+\alpha\chi_2)(\chi_1-\alpha\epsilon\chi_2)}{-k_y^2(1-\alpha)^2+\beta_0^2(\chi_1+\alpha\chi_2)(\chi_1+\alpha\epsilon\chi_2)}\right]\right\}_{k_y=0}$$
$$\times\int_{-\infty}^{+\infty}\mathrm{d}k_y\exp(-2h\chi_1). \tag{23}$$

Now integration with respect to k_y can be performed using

$$\int_{-\infty}^{+\infty}\exp[-2a(x^2+b^2)^{1/2}]\,\mathrm{d}x = (\pi b/a)^{1/2}\exp(-2ab).$$

Because of χ_2, the expression in the braces of equation (23) is complex. Substituting for χ_1, χ_2 and α, rationalising the denominator and taking only the imaginary part of the expression in the braces at $k_y = 0$, we get

$$-\partial\mathscr{E}^{\mathrm{bd}}/\partial z = \frac{e^2(1-\beta_0^2)^{5/4}(\epsilon\beta_0^2-1)^{1/2}\epsilon}{v_0^{3/2}(\pi h)^{1/2}(1-\epsilon)(1+\epsilon-\epsilon\beta_0^2)}$$
$$\times\int_{-\infty}^{+\infty}\mathrm{d}\omega\,\omega^{1/2}J_0^2(\lambda\omega)\exp[-2h(\omega/v_0)(1-\beta_0^2)^{1/2}] \tag{24}$$

where

$$\lambda = v_u(1-\beta_0^2)^{3/2}/\omega_0 v_0.$$

Here in equation (24) we have considered the situation where $\omega_0 \ll \omega_{\mathrm{m}}$ and we have therefore neglected v_u terms compared to v_0 terms. Using Luke (1962), the integral with respect to ω in equation (24) can be written as

$$\int_0^{\infty}\omega^{1/2}\exp(-\alpha_1\omega)J_0^2(\lambda\omega)\,\mathrm{d}\omega = \alpha_1^{-3/2}\sum_{m=0}^{\infty}\frac{\Gamma(2m+\frac{3}{2})}{m!\Gamma(m+1)}$$
$$\times\left(\frac{-\lambda^2}{4\alpha_1^2}\right)^m {}_2F_1(-m,-m;1;1) \tag{25}$$

where $\alpha_1 = (2h/v_0)(1-\beta_0^2)^{1/2}$ and the hypergeometric function ${}_2F_1$ is given by

$${}_2F_1(-m,-m;1;1) = \Gamma(2m+1)/\Gamma^2(m+1).$$

From equations (24) and (25) we get

$$I_{\mathrm{Cer}} = (\partial\mathscr{E}^{\mathrm{bd}}/\partial z)|_{l=0}$$
$$= \frac{e^2(1-\beta_0^2)^{1/2}(\epsilon\beta_0^2-1)^{1/2}\epsilon}{\sqrt{2\pi}h^2(\epsilon-1)(1+\epsilon-\epsilon\beta_0^2)}$$
$$\times\sum_{m=0}^{\infty}\frac{(-1)^m\Gamma(2m+\frac{3}{2})\Gamma(2m+1)}{\Gamma^4(m+1)}\left(\frac{eE_0(1-\beta_0^2)}{4m_0h\omega_0^2}\right)^{2m}. \tag{26}$$

Equation (26) gives the intensity of Čerenkov radiation due to a charge e that moves with velocity $\boldsymbol{v}_0$ in a vacuum with a relativistic superphase velocity at a height $h(h \neq 0)$

from the boundary of a dielectric when an alternating electric field is applied parallel to the boundary.

For $m=0$ equation (26) gives

$$I_0=\left.\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z}\right|_{\substack{l=0\\m=0}}=\frac{e^2(1-\beta_0^2)^{1/2}(\epsilon\beta_0^2-1)^{1/2}\epsilon}{2^{3/2}h^2(\epsilon-1)(1+\epsilon-\epsilon\beta_0^2)} \tag{27}$$

which corresponds to Čerenkov radiation in the absence of the field due to a charge e moving in close proximity to the dielectric ϵ. Equation (27) shows that the intensity of Čerenkov radiation is inversely proportional to the square of the height h at which the charge moves above the surface of a dielectric.

To estimate the relative intensities of Čerenkov radiation in the situation when the charge moves above the surface of a dielectric at a height h, and the situation when it moves through an infinite dielectric, we calculate the intensity in both cases for

$$n=\sqrt{\epsilon}=1{\cdot}495, \qquad \beta_0=0{\cdot}998, \qquad \omega_{\mathrm{m}}=3{\cdot}7674\times10^{15}\ \mathrm{Hz}.$$

From equation (27) we get

$$(I_0h^2)=0{\cdot}1463\times10^{-38}\ \mathrm{J\ m}^2$$

where h is in metres, while the well known result of Frank and Tamm (1937, 1967) gives

$$I_{\mathrm{c}}=0{\cdot}6655\times10^{-24}\ \mathrm{J}.$$

For $h=10^{-4}$ m, $I_0=0{\cdot}1463\times10^{-30}$ J, and for $h=10^{-6}$ m, $I_0=0{\cdot}1463\times10^{-26}$ J etc.

Thus, as the charge moves closer and closer to the surface of the dielectric, the intensity of Čerenkov radiation increases and becomes comparable to the intensity of Čerenkov radiation given out of an infinite isotropic dielectric. But one cannot decrease h beyond a certain limit because of technical difficulties encountered in practice. Therefore the intensity of Čerenkov radiation is at least two to three orders less in this situation than in the usual one.

The terms $m=1,2\ldots$ of equation (26) account for the effect of the parallel alternating electric field on Čerenkov radiation which is given out because of the close presence of the dielectric. The infinite sum on the right-hand side of equation (26) is convergent for

$$x'=\frac{eE_0(1-\beta_0^2)}{4m_0\omega_0^2h}<0{\cdot}25.$$

The convergence condition on x' restricts the values of the parameters E_0, ω_0, β_0 and h. We vary any one of the parameters whose effect on Čerenkov radiation is to be investigated by keeping the remaining parameters fixed. Effects of the field and the boundary can thus be investigated completely with the help of equation (26).

For $n=\sqrt{\epsilon}=1{\cdot}495$ (Perspex), $\beta_0=0{\cdot}998$, $\omega_{\mathrm{m}}=3{\cdot}7674\times10^{15}$ Hz and $\omega_0=2{\cdot}8\times10^9$ Hz, we have calculated the intensity of Čerenkov radiation in units of (I_0h^2) for various values of x' using equation (26). The variation of $(I_{\mathrm{Cer}}/I_0h^2)$ as a function of x' is shown graphically in figure 2. As $x'\alpha(x/h)$, where $x=\lambda\omega_{\mathrm{m}}$, for any fixed height h figure 2 shows that Čerenkov radiation gets reduced from its no-field value due to the presence of a parallel alternating electric field. This result is quite consistent with the conclusions drawn by Risbud and Takwale (1977) while dealing with an infinite dielectric medium. The dependence of Čerenkov radiation on height h can be studied

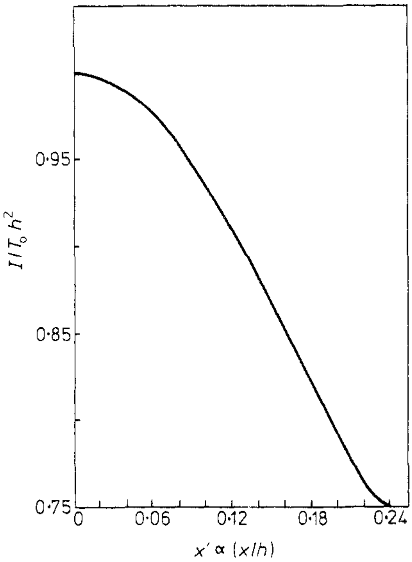

Figure 2. Variation of intensity of Čerenkov radiation with x'.

as follows. For the values of ϵ, β_0, ω_m and ω_0 given above we have

$$h = (x/x') \times 3{\cdot}1409 \times 10^{-7}\ \text{m}.$$

By fixing x one can find values of h for different values of x'. From these values of h one can calculate (I_{Cer}) in units of I_0. We have plotted the intensity of Čerenkov radiation as a function of h for $x = 1$, 10 and 100 in figure 3. Such graphs can be plotted for any

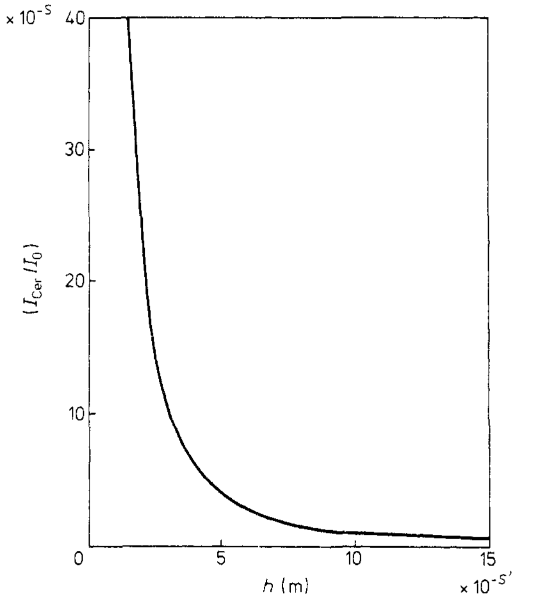

Figure 3. Variation of intensity of Čerenkov radiation with height h. For $\lambda\omega_m = 1$, $S = 10$ and $S' = -6$; for $\lambda\omega_m = 10$, $S = 8$ and $S' = -5$; for $\lambda\omega_m = 100$, $S = 6$ and $S' = -4$.

value of x of interest. Then from this graph the height h may be chosen to optimise the intensity of Čerenkov radiation.

2.6. *Radiation when the charge moves exactly along the surface of a dielectric*

Here

$$\epsilon_2 = \epsilon, \qquad \epsilon_1 = 1 = \mu_1 = \mu_2, \qquad h = 0, \qquad \epsilon\beta_0^2 > 1, \qquad l = 0.$$

Putting $h = 0$ and performing integration with respect to k_z by using a δ function in equation (22), we get

$$-\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z} = \mathrm{Re}\,\frac{ie^2}{2\pi v_0^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\omega\, dk_y \frac{\omega}{\chi_1}(1-\beta_0^2)J_0^2\left(\frac{\omega v_u(1-\beta_0^2)^{3/2}}{\omega_0 v_0}\right)$$
$$\times\left[\frac{k_y^2(1-\alpha)^2+\beta_0^2(\chi_1+\alpha\chi_2)(\chi_1-\alpha\epsilon\chi_2)}{-k_y^2(1-\alpha)^2+\beta_0^2(\chi_1+\alpha\chi_2)(\chi_1+\alpha\epsilon\chi_2)}\right]. \tag{28}$$

Consider first the integral with respect to k_y in equation (28):

$$I = \int_{-\infty}^{+\infty}\frac{dk_y}{\chi_1}\left[\frac{k_y^2(1-\alpha)^2+\beta_0^2(\chi_1+\alpha\chi_2)(\chi_1-\alpha\epsilon\chi_2)}{-k_y^2(1-\alpha)^2+\beta_0^2(\chi_1+\alpha\chi_2)(\chi_1+\alpha\epsilon\chi_2)}\right].$$

The conditions $\mathrm{Re}\,\chi_1 < 0$ and $\epsilon\beta_0^2 > 1$ restrict the region of k_y, i.e. $k_y \leq (\omega/v_0)(\epsilon\beta_0^2 - 1)$. Substituting $k_y = (\omega/v_0)\sqrt{\epsilon\beta_0^2-1}\,\xi_j$, where $\xi < 1$ always, in the integral I and putting values

$$\chi_1 = (\omega/v_0)[(\epsilon\beta_0^2-1)\xi^2+(1-\beta_0^2)]^{1/2} \qquad \chi_2 = (i\omega/v_0)\sqrt{(\epsilon\beta_0^2-1)(1-\xi^2)}$$

and $\alpha = (1-\beta_0^2)/(1-\epsilon\beta_0^2)$, we see that the expression in the square bracket is complex. Rationalising the denominator and taking only the imaginary part of the expression in the approximation $\omega_0 \ll \omega_m$, we get from equation (28)

$$-\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z} = -\frac{2e^2(\epsilon\beta_0^2-1)}{\pi v_0^2(\epsilon-1)}\int_{-\infty}^{+\infty}\omega\, d\omega J_0^2(\lambda\omega)\int_{-1}^{+1} d\xi\sqrt{1-\xi^2}\left(\frac{\xi^2+A}{\xi^2+B}\right) \tag{29}$$

where

$$A = \epsilon(1-\beta_0^2)/(\epsilon+1)(1-\epsilon\beta_0^2)$$
$$B = (1+\epsilon-\epsilon\beta_0^2)/(1+\epsilon)(\epsilon\beta_0^2-1).$$

Integration with respect to ξ in equation (29) can be performed by substituting $u = \xi/(1-\xi^2)^{1/2}$ and using the method of complex integration. The upper limit in the integral with respect to ω can be replaced by ω_m, the maximum frequency in the electromagnetic spectrum up to which the Čerenkov radiation condition can be satisfied. Integrating with respect to ξ and ω, we get from equation (29) the following expression for the intensity of Čerenkov radiation due to a charge e that moves exactly along the surface of the dielectric ϵ in the presence of a parallel alternating electric field:

$$I_{\mathrm{Cer}} = \left(\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z}\right)\Big|_{l=0}$$
$$= \frac{e^2\omega_m^2[-2\epsilon\beta_0+(1+\epsilon-\epsilon\beta_0^2)^{1/2}(1-\epsilon+\epsilon\beta_0^2+\epsilon^2\beta_0^2)]}{2v_0^2(\epsilon^2-1)(1+\epsilon-\epsilon\beta_0^2)^{1/2}}$$
$$\times[J_1^2(\lambda\omega_m)+J_0^2(\lambda\omega_m)] \tag{30}$$

where

$$\lambda = v_u(1-\beta_0^2)^{3/2}\omega_0 v_0, \qquad v_u = eE_0/m_0\omega_0$$

and J_0, J_1 are Bessel functions of the first kind.

In the absence of a field, equation (30) reduces to

$$I_0' = \frac{e^2\omega_m^2[-2\epsilon\beta_0 + (1+\epsilon-\epsilon\beta_0^2)^{1/2}(1-\epsilon+\epsilon\beta_0^2+\epsilon^2\beta_0^2)]}{2v_0^2(\epsilon^2-1)(1+\epsilon-\epsilon\beta_0^2)^{1/2}}. \tag{31}$$

A modest amount of manipulation shows that this agrees with equation (18) of Sitenko and Tkalich (1960). Calculating I_0' from equation (31) for the values of ϵ, β_0, ω_m etc as specified before in § 2.5 and comparing it with I_c (calculated in § 2.5), we see that the intensity of Čerenkov radiation when a charge moves exactly along the surface of a dielectric is about 60% of the intensity of the radiation when it moves through an infinite dielectric.

From equation (30) we can determine the intensity of Čerenkov radiation when a charge moves with a superphase velocity along the surface of a dielectric in the presence of a parallel alternating electric field of any strength and frequency in any region of interest wherever the Čerenkov radiation conditions is satisfied, and when the approximation $\omega_0 \ll \omega_m$ is valid.

3. Perpendicular field: general expression for the radiation and analysis

Let us consider the situation when an alternating electric field $\boldsymbol{E} = \boldsymbol{E}_0 \sin \omega_0 t$ is applied perpendicular to the direction of relativistic motion of the charge e, at a height h from the boundary separating two dielectrics (figure 1). Let $\boldsymbol{E}\|\boldsymbol{y}$ and $\boldsymbol{v}_0\|\boldsymbol{z}$. In this case the charge density ρ and the current density $\boldsymbol{j}$ are

$$\rho(t) = e\delta(x-h)\delta(y-y(t))\delta(z-v_0 t) \tag{32}$$

$$\boldsymbol{j}(t) = \rho(t)\boldsymbol{v}(t) \tag{33}$$

where $\boldsymbol{v}(t)$, the velocity of the charge, modified due to the presence of the perpendicular field, has the following components:

$$\begin{aligned} v_x &= 0 \\ v_y &= -v_u'(1-\beta_0^2)^{1/2}\cos\omega_0 t/[1-(v_0 v_u'/c^2)\cos\omega_0 t] \\ v_z &= v_0 \end{aligned} \tag{34}$$

and

$$y(t) = \int v_y(t)\,\mathrm{d}t.$$

We can write, in the linear approximation of v_u',

$$v_y(t) = -(1-\beta_0^2)v_u\cos\omega_0 t, \qquad v_u = eE_0/m_0\omega_0. \tag{34'}$$

Using equation (34′) we get

$$y(t) = -(v_u/\omega_0)(1-\beta_0^2)\sin\omega_0 t. \tag{35}$$

To determine the electromagnetic field of the charge, we solve Maxwell's equations with the charge and current densities given by equations (32) and (33) using the method of images, as done before for the parallel field.

3.1. *Field potentials in the upper half-space* ($x > 0$)

In the upper half-space, the electric charge density is

$$\rho_e(t) = e\delta(x-h)\delta(y-y(t))\delta(z-v_z t) + e_1(y-y(t), z-v_z t)\delta(x+h) \tag{36}$$

and the magnetic charge density is

$$\rho_m(t) = m_1(y-y(t), z-v_z t)\delta(x+h). \tag{37}$$

We find the solutions of Maxwell's equations (8) for electromagnetic potentials in the form given by equation (9). Substituting equations (36), (37) and (9) in equations (8), we obtain the following equations for the expansion coefficients $\phi(x, k_y, k_z, \omega)$, $\boldsymbol{A}(x, k_y, k_z, \omega)$, $\psi(x, k_y, k_z, \omega)$ and $\boldsymbol{F}(x, k_y, k_z, \omega)$:

$$\begin{aligned} &\partial^2\phi/\partial x^2 - \chi_1^2\phi \\ &\quad = -(e/\pi\epsilon_1)\delta(x-h)\sum_{l=-\infty}^{+\infty} J_l(k_z v_u(1-\beta_0^2)/\omega_0) \\ &\qquad \times \delta(\omega + l\omega_0 - k_z v_z) - [e_1(k_y, k_z, \omega)/2\pi^2\epsilon_1]\delta(x+h) \end{aligned} \tag{38}$$

$$A_z(x, k_y, k_z, \omega) = \beta_z\epsilon_1\mu_1\phi(x, k_y, k_z, \omega) \tag{39}$$

$$A_y(x, k_y, k_z, \omega) = \beta_y\epsilon_1\mu_1\phi(x, k_y, k_z, \omega) \tag{40}$$

$$F_y(x, k_y, k_z, \omega) = \beta_y\epsilon_1\mu_1\psi(x, k_y, k_z, \omega) \tag{41}$$

and $\psi(x, k_y, k_z, \omega)$ and $F_z(x, k_y, k_z, \omega)$ are as given by equations (12) and (13) respectively.

Here

$$\chi_1^2 = k_y^2 + k_z^2 - \omega^2 n_1^2/c^2, \qquad \beta_z = v_z/c = \beta_0, \beta_y = v_y/c,$$

$$e_1(k_y, k_z, \omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e_1(y-y(t), z-v_z t)\exp[-\mathrm{i}(k_y y + k_z z - \omega t)]\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t$$

and J_l is the Bessel function of the first kind. Now the solutions of equations (38), (39), (40), (41), (17) and (18) vanishing at infinity have the following form (using Davydov 1965):

$$\begin{aligned} &\phi^{(1)}(x, k_y, k_z, \omega) \\ &\quad = \frac{e}{2\pi\epsilon_1}\frac{\exp(-\chi_1|x-h|)}{\chi_1}\sum_{l=-\infty}^{+\infty} J_l\left(\frac{k_y v_u(1-\beta_0^2)}{\omega_0}\right) \\ &\qquad \times \delta(\omega + l\omega_0 - k_z v_z) + \frac{e_1(k_y, k_z, \omega)}{4\pi^2\epsilon_1}\frac{\exp[-\chi_1(x+h)]}{\chi_1} \end{aligned} \tag{42}$$

$$A_z^{(1)}(x, k_y, k_z, \omega) = \beta_z n_1^2\phi^{(1)}(x, k_y, k_z, \omega)$$

$$A_y^{(1)}(x, k_y, k_z, \omega) = \beta_y n_1^2\phi^{(1)}(x, k_y, k_z, \omega)$$

$$\psi^{(1)}(x, k_y, k_z, \omega) = [m_1(k_y, k_z, \omega)/4\pi^2\mu_1]\frac{\exp[-\chi_1(x+h)]}{\chi_1}$$

$$F_z^{(1)}(x, k_y, k_z, \omega) = \beta_z n_1^2 \psi^{(1)}(x, k_y, k_z, \omega)$$
$$F_y^{(1)}(x, k_y, k_z, \omega) = \beta_y n_1^2 \psi^{(1)}(x, k_y, k_z, \omega).$$

3.2. *Field potentials in the lower half-space* $(x < 0)$

In the lower half-space we determine the electromagnetic field with the aid of the electric and magnetic charge densities:

$$\rho_e(t) = e_2(y - y(t), z - v_z t)\delta(x - h)$$
$$\rho_m(t) = m_2(y - y(t), z - v_z t)\delta(x - h).$$

The expansion coefficients of the electric and magnetic potentials are

$$\begin{aligned}
&\phi^{(2)}(x, k_y, k_z, \omega) = [e_2(k_y, k_z, \omega)/4\pi^2\epsilon_2]\{\exp[-\chi_2(h - x)]/\chi_2\}\\
&A_z^{(2)}(x, k_y, k_z, \omega) = \beta_z n_2^2 \phi^{(2)}(x, k_y, k_z, \omega)\\
&A_y^{(2)}(x, k_y, k_z, \omega) = \beta_y n_2^2 \phi^{(2)}(x, k_y, k_z, \omega)\\
&\psi^{(2)}(x, k_y, k_z, \omega) = [m_2(k_y, k_z, \omega)/4\pi^2\mu_2]\{\exp[-\chi_2(h - x)]/\chi_2\}\\
&F_z^{(2)}(x, k_y, k_z, \omega) = \beta_z n_2^2 \psi^{(2)}(x, k_y, k_z, \omega)\\
&F_y^{(2)}(x, k_y, k_z, \omega) = \beta_y n_2^2 \psi^{(2)}(x, k_y, k_z, \omega)
\end{aligned} \tag{43}$$

where

$$\chi_2^2 = k_y^2 + k_z^2 - \omega^2 n_2^2/c^2, \text{ Re } \chi_2 > 0 \text{ and } x < 0.$$

3.3. *Electromagnetic field of the charge*

Using equations (42), (43) and (9) we can determine the electric potentials $\phi(\boldsymbol{r}, t)$, $\boldsymbol{A}(\boldsymbol{r}, t)$ and magnetic potentials $\psi(\boldsymbol{r}, t)$ and $\boldsymbol{F}(\boldsymbol{r}, t)$ in the two media. Electromagnetic fields can then be obtained through equations (5), (6) and (7). From the boundary conditions at $x = 0$, we determine the fictitious charge densities $e_1(k_y, k_z, \omega)$, $m_1(k_y, k_z, \omega)$, $e_2(k_y, k_z, \omega)$ and $m_2(k_y, k_z, \omega)$.

3.4. *Energy radiated*

Now as before, the energy radiated is given by

$$-\partial\mathscr{E}/\partial z = -eE_z|_{x=h, y=y(t), z=v_z t}.$$

Calculating the z-component of the electric field at the location of the point charge, we get

$$\begin{aligned}
-\frac{\partial\mathscr{E}}{\partial z} = \text{Re}\,\frac{-e^2}{2\pi}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{d}k_y\,\mathrm{d}k_z\,\mathrm{d}\omega\, J_l\left(\frac{k_y v_u(1-\beta_0^2)}{\omega_0}\right)\\
\times\left\{\frac{-ik_z}{\epsilon_1\chi_1}\left(1 - \frac{\omega v_z n_1^2}{k_z c^2}\right)\left[J_l\left(\frac{k_y v_u(1-\beta_0^2)}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_z)\right.\right.\\
\left.\left.+\frac{e_1(k_y, k_z, \omega)}{2\pi e}\exp(-2h\chi_1)\right] + \frac{v_y m_1(k_y, k_z, \omega)\exp(-2h\chi_1)}{2\pi e c}\right\}
\end{aligned} \tag{44}$$

where

$$e_1(k_y, k_z, \omega) = 2\pi e \sum_{l=-\infty}^{+\infty} J_l[k_y v_u(1-\beta_0^2)/\omega_0]\delta(\omega + l\omega_0 - k_z v_z)$$
$$\times \left\{ \frac{k_z^2[1-\alpha' - (\omega v_z/k_z c^2)(n_1^2 - \alpha' n_2^2)]^2 + \beta_y^2(\mu_1\chi_1 + \alpha'\mu_2\chi_2)(\epsilon_1\chi_1 - \alpha'\epsilon_2\chi_2)}{-k_z^2[1-\alpha' - (\omega v_z/k_z c^2)(n_1^2 - \alpha' n_2^2)]^2 + \beta_y^2(\mu_1\chi_1 + \alpha'\mu_2\chi_2)(\epsilon_1\chi_1 + \alpha'\epsilon_2\chi_2)} \right\} \quad (45)$$

with

$$\alpha' = \frac{k_z v_z[1-(\omega v_z/k_z c^2)n_1^2] + k_y v_y[1-(\omega v_y/k_y c^2)n_1^2]}{k_z v_z[1-(\omega v_z/k_z c^2)n_2^2] + k_y v_y[1-(\omega v_y/k_y c^2)n_2^2]}$$

$$m_1(k_y, k_z, \omega) = 2\mathrm{i}\mu_1\chi_1\beta_y k_z 2\pi e \sum_{l=-\infty}^{+\infty} J_l\left(\frac{k_y v_u(1-\beta_0^2)}{\omega_0}\right)$$
$$\times \left\{ \frac{\delta(\omega + l\omega_0 - k_z v_z)\left[1-\alpha' - \frac{\omega v_z}{k_z c^2}(n_1^2 - \alpha' n_2^2)\right]}{-k_z^2\left[1-\alpha' - \frac{\omega v_z}{k_z c^2}(n_1^2 - \alpha' n_2^2)\right]^2 + \beta_y^2(\mu_1\chi_1 + \alpha'\mu_2\chi_2)(\epsilon_1\chi_1 + \alpha'\epsilon_2\chi_2)} \right\}. \quad (46)$$

Equation (44) represents the total energy loss of a point charge in the presence of the perpendicular alternating electric field when the charge moves with a relativistic superphase velocity through a dielectric ϵ_1 at a height h above its boundary with dielectric ϵ_2.

Substituting $e_1(k_y, k_z, \omega)$ and $m_1(k_y, k_z, \omega)$ respectively from equations (45) and (46) in equation (44) and integrating with respect to k_z using the δ function, we get for the energy loss

$$-\frac{\partial \mathscr{E}}{\partial z} = \mathrm{Re}\frac{\mathrm{i}e^2}{2\pi v_z^2}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{d}\omega\, \mathrm{d}k_y(\omega + l\omega_0)J_l^2\left(\frac{k_y v_u(1-\beta_0^2)}{\omega_0}\right)$$
$$\times\left\{\frac{\beta_0^2 n_1^2}{\epsilon_1\chi_1}\left(\frac{1}{\beta_0^2 n_1^2} - \frac{\omega}{\omega + l\omega_0}\right)\left[1 + \frac{\exp(-2h\chi_1)}{D}\left[k_z^2\left(1-\alpha' - \frac{\omega v_z^2(n_1^2 - \alpha' n_2^2)}{(\omega + l\omega_0)c^2}\right)^2\right.\right.\right.$$
$$\left.\left.+\beta_y^2(\mu_1\chi_1 + \alpha'\mu_2\chi_2)(\epsilon_1\chi_1 - \alpha'\epsilon_2\chi_2)\right]\right]$$
$$\left.-2\beta_y^2\mu_1\chi_1\frac{\exp(-2h\chi_1)}{D}\left[1-\alpha' - \frac{\omega v_z^2(n_1^2 - \alpha' n_2^2)}{(\omega + l\omega_0)c^2}\right]\right\} \quad (47)$$

where

$$D = -k_z^2\left(1-\alpha' - \frac{\omega v_z^2(n_1^2 - \alpha' n_2^2)}{(\omega + l\omega_0)c^2}\right)^2 + \beta_y^2(\mu_1\chi_1 + \alpha'\mu_2\chi_2)(\epsilon_1\chi_1 + \alpha'\epsilon_2\chi_2).$$

The 1 in the square brackets of equation (47) agrees with equation (27) of Risbud and Takwale (1977), except for a factor $(1-\beta_0^2)$ in the argument of the Bessel function which comes in here as a correction for relativistic velocities. Also there is no distinction between v^2 and v_0^2 in equation (27) of Risbud and Takwale (1977), as stated before. This term represents the loss of energy by the charge in an unbounded dielectric ϵ_1 in a perpendicular alternating electric field. For $l = 0$ it gives Čerenkov radiation and for $l=\pm 1, \pm 2, \ldots$ it corresponds to Doppler radiation. We need not analyse it further as it is fully discussed by Risbud and Takwale (1977, 1979).

The remaining two terms containing the exponential factor $\exp(-2hx_1)$ determine the energy losses mediated by the alternating electric field. Let us consider them in the following two interesting situations.

3.5. *Radiation when the charge moves in a vacuum at a height h from the surface of a dielectric*

Here $\epsilon_1=\mu_1=\mu_2=1$, $\epsilon_2=\epsilon$, $h\neq 0$, $\epsilon\beta_0^2>1$, $l=0$. Consider integration with respect to k_y in equation (47). Here the exponential factor dominates over the whole expression and, as discussed before, we can keep it inside the integral and take the remaining slowly varying function of k_y outside the integral at the value $k_y=0$. Thus we can write from equation (47)

$$-\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z}=\operatorname{Re}\frac{ie^2}{2\pi v_z^2}\int_{-\infty}^{+\infty}\omega\,\mathrm{d}\omega\, J_0^2(0)\left[\int_{-\infty}^{+\infty}\mathrm{d}k_y\exp(-2h\chi_1)\right]$$
$$\times[\![[(1-\beta_z^2)/D']\{(\omega^2/v_z^2)[1-\beta_z^2-\alpha'(1-\epsilon\beta_z^2)]^2$$
$$+\beta_y^2(\chi_1+\alpha'\chi_2)(\chi_1-\alpha'\epsilon\chi_2)\}$$
$$-(2\beta_y^2\chi_1^2/D')[1-\beta_z^2-\alpha'(1-\epsilon\beta_z^2)]]\!]|_{k_y=0} \tag{48}$$

where

$$D'=\chi_1\{(-\omega^2/v_z^2)[1-\beta_z^2-\alpha'(1-\epsilon\beta_z^2)]^2+\beta_y^2(\chi_1+\alpha'\chi_2)(\chi_1+\alpha'\epsilon\chi_2)\}.$$

Simplifying, keeping terms up to orders of β_y^4 and taking the imaginary part of the expression in the shadow brackets at $k_y=0$, and performing integrations with respect to k_y and ω in equation (48), we get the following result:

$$I_{\mathrm{Cer}}=\left.\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z}\right|_{l=0}=\frac{e^2\epsilon(1-\beta_z^2)^{1/2}(\epsilon\beta_z^2-1)^{1/2}}{2^{3/2}h^2(\epsilon-1)(1+\epsilon-\epsilon\beta_z^2)}. \tag{49}$$

Equation (47) represents Čerenkov radiation in a perpendicular alternating electric field due to a charge e that moves in vacuum with a relativistic velocity satisfying the condition $\epsilon\beta_0^2>1$ at a height h from the surface of a dielectric ϵ. As field parameters do not enter in equation (49), the field does not affect the radiation, at least in the linear approximation of v_u. Equation (49) agrees with the no-field case represented by equation (27) obtained for the parallel field.

3.6. *Radiation when the charge moves exactly along the surface of a dielectric*

Here

$$\epsilon_1=\mu_1=\mu_2=1, \qquad \epsilon_2=\epsilon, \qquad h=0, \qquad \epsilon\beta_0^2>1, \qquad l=0.$$

Substituting $k_y=(\omega/v_z)(\epsilon\beta_z^2-1)^{1/2}\xi$ where $\xi<1$, in the second and third terms of equation (47), simplifying and taking terms up to orders β_y^4, we get the following equation for energy loss:

$$-\frac{\partial \mathscr{E}^{\mathrm{bd}}}{\partial z}=\operatorname{Re}\frac{-2e^2(\epsilon\beta_z^2-1)}{\pi v_z^2(\epsilon-1)}\int_0^{\infty}\omega\,\mathrm{d}\omega\int_{-1}^{+1}\mathrm{d}\xi\, J_0^2(\lambda'\xi)\sqrt{1-\xi^2}\left(\frac{\xi^2+A}{\xi^2+B}\right) \tag{50}$$

where

$$A = \frac{\epsilon(1-\beta_z^2)}{(1+\epsilon)(\epsilon\beta_z^2-1)}, \qquad B = \frac{1+\epsilon-\epsilon\beta_z^2}{(1+\epsilon)(\epsilon\beta_z^2-1)}$$

and

$$\lambda' = (\omega v_u/\omega_0 v_z)\sqrt{\epsilon\beta_z^2-1}(1-\beta_z^2).$$

Consider the integral with respect to ξ in equation (50). Substituting $\xi = \sin\theta$, it can be written as

$$I = \int_{-1}^{+1} \mathrm{d}\xi\, J_0^2(\lambda'\xi)\sqrt{1-\xi^2}\left(\frac{\xi^2+A}{\xi^2+B}\right)$$

$$= \int_{-\pi/2}^{+\pi/2} J_0^2(\lambda'\sin\theta)\cos^2\theta\,\mathrm{d}\theta + (B-A)\int_{-\pi/2}^{+\pi/2} J_0^2(\lambda'\sin\theta)\,\mathrm{d}\theta$$

$$+ (A-B)(1+B)\int_{-\pi/2}^{(p/2} \frac{J_0^2(\lambda'\sin\theta)\,\mathrm{d}\theta}{\sin^2\theta+B}. \tag{51}$$

Using Gradshteyn and Ryzhik (1965) we see that the first two integrals in equation (51) are special cases of the following integral:

$$\int_0^{\pi/2} J_\mu(z\sin t)J_\nu(z\sin t)\sin^{2\alpha-1}t\cos^{2\beta-1}t\,\mathrm{d}t$$

$$= \frac{1}{2}\frac{(z/2)^{\mu+\nu}\Gamma(\beta)\Gamma[\frac{1}{2}(\mu+\nu)+\alpha]}{\Gamma(\mu+1)\Gamma(\nu+1)\Gamma[\frac{1}{2}(\mu+\nu)+\alpha+\beta]}$$

$$\times {}_3F_4\left(\begin{matrix}\frac{1}{2}(\mu+\nu+1), \frac{1}{2}(\mu+\nu+2), \frac{1}{2}(\mu+\nu)+\alpha \\ \mu+\nu+1, \mu+1, \nu+1, \frac{1}{2}(\mu+\nu)+\alpha+\beta\end{matrix}\middle| -z^2\right)$$

where

$$\mathrm{Re}(\mu+\nu+2\alpha) > 0, \qquad \mathrm{Re}\,\beta > 0$$

and ${}_3F_4$ denotes a hypergeometric function.

The last integral in equation (51) can be evaluated by substituting $z = \exp(\mathrm{i}\theta)$ and using the method of complex integration. Thus we get for equation (51)

$$I = \sum_{n=0}^{\infty} [(-1)^n\Gamma^2(n+\tfrac{1}{2})/\Gamma^4(n+1)][B-A+1/2(n+1)](\lambda'\omega)^{2n}$$

$$+ \pi(A-B)(1+1/B)^{1/2}I_0^2(\lambda'\sqrt{B}\omega). \tag{52}$$

Substituting the value of the integral I from equation (52) in equation (50) and integrating with respect to ω by replacing the upper limit ∞ by ω_{m} and using Gradshteyn and Ryzhik (1965), we get finally

$$I_{\mathrm{Cer}} = (\partial\mathscr{E}^{\mathrm{bd}}/\partial z)|_{t=0}$$

$$= \frac{e^2(\epsilon\beta_z^2-1)\omega_{\mathrm{m}}^2}{\pi v_z^2(\epsilon-1)}\left\{\frac{\pi\epsilon\beta_z[I_1^2(\lambda''\omega_{\mathrm{m}}) - I_0^2(\lambda''\omega_{\mathrm{m}})]}{(1+\epsilon)(\epsilon\beta_z^2-1)\sqrt{1+\epsilon-\epsilon\beta_z^2}}\right.$$

$$\left. + \sum_{n=0}^{\infty}\frac{(-1)^n\Gamma^2(n+\frac{1}{2})}{(n+1)\Gamma^4(n+1)}\left[\frac{1}{2(n+1)} + \frac{1}{(1+\epsilon)(\epsilon\beta_z^2-1)}\right](\lambda'\omega_{\mathrm{m}})^{2n}\right\} \tag{53}$$

where

$$\lambda' = \lambda(\epsilon\beta_z^2 - 1)^{1/2}, \qquad \lambda'' = \lambda\sqrt{\frac{1+\epsilon-\epsilon\beta_z^2}{1+\epsilon}}, \qquad \lambda = \frac{v_u}{\omega_0 v_z}(1-\beta_z^2)$$

and I_0, I_1 are modified Bessel functions of the first kind.

Equation (53) gives Čerenkov radiation in the presence of a perpendicular alternating electric field when a charge moves with a constant relativistic superphase velocity exactly along the surface of a dielectric. In the absence of a field, equation (53) reduces to the no-field case: equation (31) obtained for the parallel field. The infinite sum appearing on the right-hand side of equation (53) is absolutely convergent. From equation (53) one can evaluate the intensity of Čerenkov radiation in this situation for any values of field parameters, for any frequency region of interest wherever the Čerenkov radiation condition is satisfied and the approximation $\omega_0 \ll \omega_m$ is valid.

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